行政院國家科學委員會專題研究計畫　成果報告

隨機移除之逐步型二設限樣本及一般化逐步型二設限樣本對極值分配雙參數之區間估計

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隨機移除之逐步型二設限樣本及一般化逐步型二設限樣本對極值分配雙參數之區間估計

Interval Estimation of Parameters of Extreme Distribution for Type II Progressive Censored Samples or General Type II Progressive Censored Samples

計劃編號: NSC92-2118-M-032-007
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主持人: 吳淑妃 淡江大學統計系

一. 中文摘要

對一組樣本大小為 n 的樣本，由於時間、成本的限制或資料蒐集時人為的疏失，而無法取得完整樣本，而取得的部分樣本稱為設限樣本. 而逐步型二設限樣本就是其中一種，一般完整樣本的統計分析方法並無法適用. 近來如 Lawless(1982), Cohen(1991) 等已開始探討對設限樣本之統計分析方法. 在醫學的臨床實驗中，有的病人在實驗未結束之前，可能因為某些因素而無法繼續實驗，又如在實驗中可能有一些實驗者對實驗有危險性，或不適合實驗的要求，而要提前結束實驗，為了解決這些問題，所以發展出二項與隨機移除型二逐步設限的方法。

在這篇報告裡，我們提出了隨機移除之逐步型二設限樣本及一般化逐步型二設限樣本對極值分配雙參數之區間估計. 在本研究中，對隨機移除之逐步型二設限樣本，設只觀察到 m 個設限樣本，我們提出 J,(J=1,…,m-1)個樞紐量，針對雙參數之型 I 極值分配(極小值之極值分配)的參數做區間估計; 對隨機移除之一般化逐步型二設限樣本，設左設限個數為 r 個，我們則提出了 J,(J=r+1,…,m-1)個樞紐量，針對雙參數之型 I 極值分配(極小值之極值分配)的參數做區間估計。而利用類似的方法，我們最後亦可推得極大值之極值分配的區間估計方法。在二項隨機移除(Binomial Removal)之不同(n.m,p)組合下，以及均勻移除(Uniform Removal(PCR))之不同(n.m)組合下，以最短平均信賴區間長度，以及最小平均聯合信賴區域面積為選取最佳樞紐量之準則. 最後並會給一個數值實例以示範所有區間估計結果.

關鍵辭:
隨機移除之逐步型二設限; 隨機移除之一般化逐步型二設限; 型 I 極值分配; 極大值之極值分配; 信賴區間; 聯合信賴區域.

ABSTRACT

Studies in the lifetimes of the organisms and products are often the main research topics industries. The past research developed some censored methods. Before the end of experiments, some patients might not proceed the experiments due to some factors. The Type II progressive censoring with random removals is thus arisen.

We propose m-1 pivotal quantities to construct the confidence interval ,confidence region for the

parameters of Type I Extreme Value Distribution for Type II progressive censored sample and generally Type II progressive censored sample under binomial and random removals separately, where m is the number of observations for Type II progressive censored sample. Finally, we compare the performances of the proposed pivotal quantities based on the length of confidence interval and the area of confidence region.

Key Words and Phrases: Type II progressive censoring with random removals; Generally Type II progressive censoring with random removals; Type I Extreme-Value Distribution;

對一組樣本大小為n的樣本，由於時間、成本的限制或資料蒐集時人為的疏失，而無法取得完整樣本，而取得的部分樣本稱為設限樣本.近來如Lawless(1982), Cohen(1991)等已開始探討對設限樣本之統計分析方法.在可靠度分析(Analysis of Reliability) 及壽命檢定實驗(Life Test Experiments)方面, see, e.q., pp. 220-221 of Johnson and Kotz (1970)就常有此種設限的問題產生.

本研究是針對假設 X 為具(型Ⅰ)極值分配參數為β與θ之一隨機變數，其中θ為尺度參數(scale parameter)而β為位置參數(location parameter)，且 X 機率密度函數(probability density function)為：

$$f(x)=\frac{1}{\theta}\exp\left[\frac{x+\beta}{\theta}-\exp\left(\frac{x+\beta}{\theta}\right)\right],-\infty\le x\le\infty,-\infty\le\beta\le\infty,\theta>0$$

；

其累積分配函數(cumulative distribution function)為

$$F(x)=1-\exp\left[-\exp\left(\frac{x+\beta}{\theta}\right)\right]。$$

設第 1 個失敗的產品有被觀察到，記為 X_1，以此類推，觀察到的是第 i 個有序樣本，記為 X_i，則隨機移除 R_i 個隨機樣本. $i=1,\ldots,m$，且設限計劃(Censoring Scheme) $\boldsymbol{R}$=(R_1=r_1,…,R_{m-1}=r_{m-1})是事先決定的, where $R_m=n-r_{r+1}-\ldots-r_{m-1}-m$.則 $X_1<X_2<\Lambda<X_m$ 為兩參數β、θ之極值分配在型二逐步設限下的有序樣本，此處 m 是事先決定的。在型二逐步設限下 $\boldsymbol{R}$=(R_1=r_1,…,R_{m-1}=r_{m-1})是隨機的包括

(1). **二項移除**

$$P\left(R_i=r_i\middle|R_{i-1}=r_{i-1},\Lambda,R_1=r_1\right)=\binom{n-m-\sum_{j=1}^{i-1}r_j}{r_i}p^{r_i}(1-p)^{n-}$$

，

上式中

$$0\le r_i\le n-m-(r_1+\Lambda+r_{i-1}),\ i=2,\Lambda,m-1$$

。

(2) **均勻移除**

$$P\left(R_i=r_i\middle|R_{i-1}=r_{i-1},\mathrm{K},R_1=r_1\right)=\frac{1}{n-m-(r_1+\Lambda+r_{i-1})+1}$$

，

上式中

$$0\le r_i\le n-m-(r_1+\Lambda+r_{i-1}),\quad i=1,\mathrm{K},m-1$$

。

(a) **隨機移除之逐步型二設限樣本下雙參數β與θ的信賴區間**

設 $X_1<X_2<\Lambda<X_m$ 為 n 個隨機樣本中的前 m 個失敗的有序樣本，此處 m 是事先決定的。則 $Y_i=\exp\left(\frac{X_i+\beta}{\theta}\right)$ 是尺度參數為 1 之指數分配在型二逐步設限下的有序樣本，當 $\boldsymbol{R}$=(R_1=r_1,…,R_{m-1}=r_{m-1})事先固定時，利用變數轉換(參考 Thomas, D. R. & Wilson, W. M. (1972))：

$$
\begin{cases}
Z_1 = nY_1, \\
Z_2 = (n - r_1 - 1)(Y_2 - Y_1), \\
Z_3 = (n - r_1 - r_2 - 2)(Y_3 - Y_2), \\
\Lambda \quad \Lambda \\
Z_m = (n - r_1 - \Lambda - r_{m-1} - m + 1)(Y_m - Y_{m-1})
\end{cases}
$$

(*)

可以得到$Z_1,\ldots,Z_m$為 iid 之標準指數分配. 故可得到下列輔助定理：

[輔助定理] 若令$U_j = 2\sum_{i=1}^{j} Z_i$，

$V_j = 2\left(\sum_{i=j+1}^{m} Z_i\right)$，則

(a) $U_j \sim \chi^2(2j)$，

(b) $V_j \sim \chi^2(2(m-j))$，

(c) U_j與V_j獨立。

我們提出 j,(j=1,⋯,m-1)個樞紐量：

$$h_j = \frac{jV_j}{(m-j)U_j} = \frac{j}{m-j}$$

$$\frac{-(n - r_1 - \ldots - r_{j-1} - j)Y_j + \sum_{i=j+1}^{m}(r_i + 1)Y_i}{\sum_{i=1}^{j}(r_i + 1)Y_i},$$

$$g = (U_j + V_j) =$$

$$2\left[\sum_{i=1}^{m-1}(r_i + 1)Y_i + (n - r_1 - \ldots - r_{m-1} - m + 1)Y_m\right]$$

樞紐量h_j與g互相獨立且

$h_j \sim F(2(m-j), 2j)$且$g \sim \chi^2(2m)$。

我們有下面兩個區間估計之定理：

[定理 1]

$$[h_j^{-1}\left(X_1, \Lambda, X_m; F_{\frac{\alpha}{2}}(2(m-j), 2j)\right) < \theta < h_j^{-1}\left(X_1, \Lambda, X_m; F\right.$$

為參數θ的$1-\alpha$水準之信賴區間。

[定理 2]

$$
\begin{cases}
h_j^{-1}[X_1, \Lambda, X_m; F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j), 2j)] < \theta < h_j^{-1}[X_1, \Lambda, X \\
g^{-1}[X_1, \Lambda, X_m; \chi^2_{\frac{1+\sqrt{1-\alpha}}{2}}(2m)] < \beta < g^{-1}[X_1, \Lambda, X_m; \chi^2_{1-}
\end{cases}
$$

為參數β和θ的$1-\alpha$水準之聯合信賴區間。

(b) 隨機移除之一般化逐步型二設限樣本下雙參數β與θ的信賴區間

設前 r 個失敗的有序樣本沒被觀察到，第一次觀察到的是第 r+1 個有序樣本，記為X_{r+1}，以此類推，觀察到第$(r+i)$個有序樣本，記為X_{r+i}，則隨機移除R_{r+i}個隨機樣本. $1 \le i \le m-r$. 則$X_{r+1} < \Lambda < X_m$稱為一般化逐步型二設限樣本，且設限計劃(Censoring Scheme)為 $R_r = (R_{r+1} = r_{r+1}, \ldots, R_{m-1} = r_{m-1}, R_m = r_m)$ 是事先決定的，where $R_m = n - r_{r+1} - \ldots - r_{m-1} - m$. 當$r = 0$時，一般化逐步型二設限樣本即為前面之型二設限樣本. 我們提出 j,(j=r+2,⋯,m-1)個樞紐量：

$$h_{2j} = \frac{(j-r-1)V_j}{(m-j)U_j} = \frac{j-r-1}{m-j}\frac{2\sum_{i=j+1}^{m} Z_i}{2\sum_{i=r+2}^{j} Z_i},$$

$$g_2 = 2\sum_{i=r+2}^{m} Z_i .$$

樞紐量h_{2j}與g_2互相獨立且

$h_{2j} \sim F(2(m-j), 2(j-r-1))$且

$g_2 \sim \chi^2(2(m-r-1))$。

我們有下面兩個區間估計之定理：

[定理 3]

$$[h_{2j}^{-1}\left(X_{r+1},\Lambda,X_m;F_{\frac{\alpha}{2}}(2(m-j),2(j-r-1))\right)$$

$$<\theta<h_{2j}^{-1}\left(X_{r+1},\Lambda,X_m;F_{1-\frac{\alpha}{2}}(2(m-j),2(j-r))\right)]$$

為參數θ的$1-\alpha$水準之信賴區間。

[定理 4]

$$\begin{cases} h_{2j}^{-1}[X_1,\Lambda,X_m;F_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-j),2(j-r-1))]< \\ \theta<h_{2j}^{-1}[X_1,\Lambda,X_m;F_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-j),2(j-r))] \\ g_2^{-1}[X_1,\Lambda,X_m;\chi^2_{\frac{1+\sqrt{1-\alpha}}{2}}(2(m-r-1))]<\beta \\ <g_2^{-1}[X_1,\Lambda,X_m;\chi^2_{\frac{1-\sqrt{1-\alpha}}{2}}(2(m-r))] \end{cases}$$

為參數β和θ的$1-\alpha$水準之聯合信賴

區間。

為數組結果與分析:下表是 n=10,m=8,9,10,r=0,1,均勻移除(PCR)和二項移除(P=.1,.3,.5,.8)

之平均信賴區間長度和平均信賴區域面積(模擬次數 5000 次), 在給定(n,m,P)之下(*)表最佳樞紐量(最佳 j 值)

r=0			PCR	P=0.1	P=0.3	P=0.5	P=0.8
n=10	m=8	J=1	0.01927	0.0212	0.02023	0.01982	0.01953
			0.00074	0.00088	0.00081	0.00079	0.00076
		J=2	0.01755*	0.01946*	0.01826*	0.01788*	0.01811
			0.00063*	0.00076*	0.00069*	0.00066*	0.00068
		J=3	0.01815	0.02106	0.0189	0.01875	0.01801*
			0.00067	0.00091	0.00073	0.00072	0.00067*
		J=4	0.02014	0.02366	0.02147	0.0202	0.01998
			0.00084	0.00114	0.00097	0.00085	0.00081
		J=5	0.02437	0.02924	0.0259	0.02476	0.02506
			0.00124	0.0018	0.00142	0.00133	0.00132
		J=6	0.03672	0.04681	0.03963	0.03913	0.03716
			0.00306	0.00549	0.00381	0.00371	0.00324
		J=7	0.15193	0.21535	0.15537	0.15134	0.14651
			0.11023	0.23509	0.11482	0.1079	0.09805
	m=9	J=1	0.01898	0.01972	0.01916	0.01885	0.01855
			0.00067	0.00071	0.00068	0.00067	0.00064
		J=2	0.01699*	0.01808	0.01672*	0.01729	0.01677*
			0.00055*	0.00063	0.00054*	0.00058	0.00055*
		J=3	0.01719	0.01778*	0.01717	0.01678*	0.01696
			0.00056	0.00061*	0.00057	0.00054*	0.00056
		J=4	0.01823	0.01887	0.01861	0.01802	0.0188

			0.00063	0.00069	0.33367	0.00064	0.00069
		J=5	0.02032	0.02173	0.02046	0.0207	0.02042
			0.00081	0.00091	0.00081	0.00085	0.0008
		J=6	0.0251	0.02722	0.02635	0.02527	0.02541
			0.00127	0.00147	0.00137	0.00125	0.00127
		J=7	0.03875	0.04226	0.04054	0.03964	0.03751
			0.00334	0.00404	0.00372	0.00345	0.00352
		J=8	0.15237	0.1871	0.15486	0.15934	0.15357
			0.10766	0.17401	0.10633	0.11479	0.10658
	m=10	J=1	0.0183	0.01835	0.0186	0.01844	0.01846
			0.00059	0.00059	0.00061	0.0006	0.0006
		J=2	0.01623	0.01653	0.01613	0.01589	0.01637
			0.00048	0.00051	0.00048	0.00046	0.00049
		J=3	0.0157*	0.01603*	0.01601*	0.01576*	0.01592*
			0.00045*	0.00048*	0.00048*	0.00046*	0.00047*
		J=4	0.01687	0.01667	0.01672	0.01722	0.01655
			0.00052	0.00052	0.00051	0.00055	0.00051
		J=5	0.01826	0.01792	0.01852	0.01851	0.01827
			0.00061	0.00058	0.00063	0.00063	0.00062
		J=6	0.02055	0.02073	0.0214	0.0209	0.02022
			0.00077	0.0008	0.00083	0.0008	0.00076
		J=7	0.02629	0.02542	0.02602	0.02592	0.02549
			0.00138	0.00121	0.00126	0.00127	0.00122
		J=8	0.04158	0.04032	0.039	0.04086	0.03989
			0.00371	0.00353	0.00334	0.00348	0.00342
		J=9	0.1647	0.16066	0.17487	0.1639	0.17143
			0.11468	0.10649	0.12766	0.11558	0.12858
r=1							
n=10	m=8	J=3	0.0205	0.01991*	0.01958*	0.02041	0.02051*
			0.0923	0.0957*	0.08675*	0.08377	0.07862*
		J=4	0.02227	0.02024	0.0216	0.02287	0.0226
			0.17269	0.12056	0.12789	0.16991	0.16127
		J=5	0.02254	0.01991	0.02183	0.0223	0.02383
			0.08574*	0.16492	0.08803	0.06351*	0.14553
		J=6	0.01983*	0.02044	0.02031	0.0203*	0.02081
			0.13239	0.20381	0.09343	0.11044	0.17933
	m=9	J=3	0.2092*	0.02043*	0.02079*	0.02049*	0.02069*

		0.07919	0.07798	0.0741	0.0685	0.07296*
	J=4	0.02224	0.02167	0.02403	0.02335	0.02417
		0.08337	0.04136*	0.05108*	0.09621	0.12115
	J=5	0.02345	0.02343	0.02304	0.02367	0.02341
		0.04774*	0.12947	0.09317	0.09917	0.09267
	J=6	0.0222	0.02274	0.0228	0.02273	0.022407
		0.09422	0.14101	0.09679	0.05344*	0.12561
	J=7	0.02134	0.02059	0.02132	0.02134	0.02091
		0.14685	0.10902	0.12175	0.22951	0.17276
m=10	J=3	0.02019*	0.2011*	0.01998*	0.01977*	0.02105*
		0.05727*	0.05439*	0.05777*	0.05967*	0.05669
	J=4	0.02453	0.02256	0.02426	0.02316	0.02232
		0.13197	0.08868	0.12747	0.09488	0.21561
	J=5	0.02388	0.02413	0.02522	0.02479	0.02384
		0.12802	0.07781	0.06719	0.08134	0.10012
	J=6	0.02512	0.02543	0.02382	0.02374	0.02466
		0.13756	0.07806	0.0595	0.0849	0.02389*
	J=7	0.02442	0.02293	0.02442	0.02388	0.02328
		0.08942	0.08345	0.10714	0.12262	0.12138
	J=8	0.02144	0.02201	0.0212	0.02169	0.02159
		0.08666	0.16925	0.1244	0.17134	0.11764

數值實例示範

With Binomial random censoring with P=.1, for(n,m,P)=(10,8,.1), the sample is given by (X_1, Λ, X_8) = (-.03595 ,-.02679, -.01604 ,-.01216, -.01165 ,-.01094, -.00691, -.00414). As to the 95% confidence interval for θ, for J=1, we need the tables values $F_{.025}(14,2)$ =.20590 and $F_{.975}(14,2)$ =39.42650. The 95% confidence intervals for θ is thus given by (.00484,.02951) with the confidence lengh .02466. As to the 95% joint confidence region for θ and β, we need the following table values $F_{.0127}(14,2)$ = .16482, $F_{.9873}(14,2)$ =78.41471, $\chi^2_{.0127}(16)$ = 6.06839, $\chi^2_{.9873}(16)$ =31.20696. The joint confidence region is given by the following inequality:

$$\begin{cases} .00435 < \theta < .03481 \\ \theta \ln \dfrac{6.06839}{2\sum_{i=1}^{m}(r_i+1)\exp\left(\dfrac{X_i}{\theta}\right)} < \beta < \theta \ln \dfrac{31.20696}{2\sum_{i=1}^{m}(r_i+1)\exp\left(\dfrac{X_i}{\theta}\right)} \end{cases}$$

The area of the confidence region is given by .00098. Similarly, the length of confidence intervals and the areas of the confidence regions for different J under Binomial removals with various probability P and uniform removals (PCR) are listed in Table 1. The optimal pivotal quantity (j value) is marked by an asterisk(*) for the given (n,m,P).

Table 1. The length of 95% confidence intervals for θ and the area of 95% confidence region for θ and β (n=10,m=8, (*) stand for the optimal one.)

P=.1	J	length	area
	1	.02466	.00098
	2	.03096	.00162
	3	.02387	.00107
	4	.03270	.00325
	5	.02192(*)	.00084(*)
	6	.05239	.00521
	7	.17327	.09724
P=.3	1	.03222	.00171
	2	.01863	.00061
	3	.02349	.00108
	4	.01335（*	.00031 (*)
	5	.01561	.00043
	6	.01373	.00035
	7	.10657	.03636
P=.5	1	.05353	.00470
	2	.02594	.00138
	3	.03165	.00195
	4	.05514	.00946
	5	.02264(*)	.00095(*)
	6	.04301	.00361
	7	.25829	.21695
P=.8	1	.02025	.00071
	2	.01904(*)	.00064(*)
	3	.02401	.00102
	4	.03796	.00249
	5	.03394	.00204
	6	.05564	.00592
	7	.24895	. 20036
PCR	1	.03381	.00193
	2	.03712	.00243
	3	.02389	.00101
	4	.02346(*)	.00098（*）
	5	.04387	.00338
	6	.03115	.00181
	7	.25303	.20551

由於篇幅限制只能些結果，其他結果保留在作者處

四．計劃結果與自評

我已完成理論推導以及模擬的研究，所有的程式皆已建立完成．故此研究已完成百分之百．此研究結果相信很快就可在學術學刊發表．

五．參考資料

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